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OPTIMAL TAX POLICY AND EXTERNALITY WITH GENERAL EQUILIBRIUM EFFECTS*

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Absract: This study analyses the effect of pollution to the optimal taxation and public provision. Environmental deterioration is modelled to be created by a harmful externality of consumption. It is assumed that there are two types of households with high and low productivity, endogenous wages and a government using a mixed taxation scheme. As a result of the asymmetric information the government needs to take the self-selection constraint into account when designing the optimal tax policy. It will be shown that the social valuation of the externality consist of terms indicating the effects directed on consumers, producers, the government and the labour markets respectively. It turns out that in the consumer side the direct effect and the self-selection effect have opposite signs indicating that the environmental and the redistributive objectives are inconsistent. However, in the producer side the direct effect and the indirect effect from wage adjustment in the labour markets have both positive signs indicating consistence between these two goals of the government. Another considerable result is that endogeneity does not change optimal commodity taxation and Dixit's principle of targeting continues to hold.

Keywords: Optimal income taxation, commodity taxation, public provision, externality, endogenous wages.

JEL-classification: D82, H21, H23

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1. Introduction

Imposing a tax on an environmentally harmful commodity is widely known to be beneficial from an efficiency point of view. According to Sandmo (1975), commodity tax on a good creating a harmful externality should appear additively only in the tax rate of that good, not in other commodities' tax rates. This is a so called additivity property, which was later shown to be a special case of Dixit's principle of targeting (Dixit, 1985). Imposing a tax on an environmentally harmful good is argued to create a double dividend, i.e. in addition to diminishing pollution, the tax revenues can be used to replace more distorting taxes and thus improve economy's efficiency. An excellent survey on the double dividend created by environmental taxation is presented in Goulder (1995).

There are two important aspects the government needs to take into account when designing the optimal tax policy for environmentally harmful goods: efficiency and income distribution. The efficiency objective is examined in a representative consumer economy e.g. in Bovenberg and van der Ploeg (1994) and Bovenberg and de Mooij (1994). The result that environmental taxes tend to be regressive (Smith, 1992; Harrison, 1994) draws attention to redistributional issues. The first study combining environmental taxation with redistributional objectives is Sandmo (1975), which analysed the structure of commodity taxation. This issue is discussed also in Pirttilä and Schöb (1996), who use an assumption of optimal linear taxation in their work.

Instead of linear taxation it is more realistic to assume a non-linear income tax and linear commodity tax. This mixed taxation case used in this work is introduced in Mirrlees (1976) and Atkinson and Stiglitz (1976). Environmental and redistributional objectives are combined in a mixed tax framework in Pirttilä and Tuomala (1997).

This study combines two usual market failures, externalities and imperfect information. It is based on a model of a two type economy, first introduced in Stiglitz (1982) and Stern (1982). This framework was developed further in Boadway and Keen (1993) and Edwards, Keen and Tuomala (1994). There are two different types of households varying by their abilities and the government needs to take the incentive constraint into account in designing the tax scheme.

The extension used in this study concerns factor price determination. Instead of assuming that wages are determined ex ante here wages are let to adjust endogenously. This assumption turns out to have an essential effect on well known results like Atkinson's and Stiglitz's result of redundancy of commodity taxation or Diamond's and Mirrlees's result of production efficiency. Both these results no longer hold when endogenous wages are introduced (see e.g. Naito, 1999; Gaube, 2001; Micheletto, 2001).

The aim of this study[1] is to analyse the optimal tax policy in the presence of an externality. In the background there are assumptions that the government aims at small income differences and a clean environment and it is restricted by self-selection constraint. The model is similar to the one in Edwards, Keen and Tuomala (1994) and Pirttilä and Tuomala (1997) with a distinction of an assumption of endogenous wages.

The paper is constructed as follows. In section 2 the model and the maximization problem are introduced. Also the first order conditions defining the optimal tax policy are calculated. In section 3 the social valuation of the externality is defined. There appears to be a controversy between environmental and redistributional objectives in the terms indicating the externality's effect on consumers. However the term demonstrating the impact of the externality on the labour markets (i.e. indirectly on both consumers and producers) implies that both of the government's goals are in accordance. Section 4 the marginal effective tax rates for both types of households are determined. It turns out that they differ from the ones in the exogenous wage case only by a couple of extra terms indicating the indirect effect of the externality on the wage rate. The optimal commodity tax is calculated in section 5. It is shown that Dixit's "principle of targeting" continues to hold despite of the assumption of endogenous wages. In section 6 there is a brief analysis of the externality's effect on public good provision. Finally section 7 concludes.

[1] This is a preliminary version of the study, a more extensive analysis is coming soon.

2. The Model

In this framework there are assumed to be two types of households with similar preferences but different productivities. Type 1 households have lower productivity and type 2 households supply labour with higher productivity. Wages are assumed to be determined endogenously, although without a loss of generality it can be assumed that $w^2 > w^1$, i.e. the wage rate of a high productivity type household is greater than the wage rate of low productivity households. The wage rate ratio is defined as $\Omega = \frac{w^1}{w^2}$.

The households earn a labour income equal to $Y^h = w^h L^h$, where L^h denotes the labour supply of type h, $h = 1,2$. Labour income is taxed by an optimal, non-linear income tax scheme $T(Y^h)$ such that the after-tax income is $B^h = Y^h - T(Y^h)$. The whole after-tax income is consumed on two goods X_i, $I = c,d$. The goods are produced with a production function $F(L^1, L^2, G, E)$[2], where $\frac{\partial F}{\partial L^h} = w^h > 0$, $\frac{\partial F}{\partial G} > 0$, G denotes the public good and $\frac{\partial F}{\partial E} < 0$. There is also a commodity tax t_i, $i = c,d$ such that the consumer prices are denoted by a vector $\underline{Q}$, where $\underline{Q} = \begin{bmatrix} q_c \\ q_d \end{bmatrix} = \begin{bmatrix} p_c \\ p_d \end{bmatrix} + \begin{bmatrix} t_c \\ t_d \end{bmatrix}$. The constraint that households use all their net income into consumption implies that $\sum_i q_i X_i^h = B^h$, $i = c,d$. Consumers' maximization of direct utility U conditional on budget constraint defines the indirect utility function $V^h(\underline{Q}, B^h, L^h, G, E) = \max_X \left\langle U\left(X, \frac{Y}{w}, G, E\right) \middle| \sum_i q_i X_i = B \right\rangle$, where E denotes the externality and G is the public good.

Good X_c is a "clean" good, whereas good X_d is a "dirty" good creating a harmful externality on the environment. The source of the externality E is the aggregate consumption of the dirty good,

[2] The production function is assumed to have constant returns of scale and it is also assumed to be possible to tax the profits with 100 percent tax rate.

$$E = \sum_h X_d^h\left(\underline{Q}, B^h, L^h, G, E\right). \qquad \textbf{eq. 1}$$

In this framework the government produces a public good *G* and collects tax revenues from a non-linear income tax and linear commodity taxation. The planner has both environmental and redistributive objectives. The governments' budget constraint[3] requires that

$$\begin{aligned} &\sum_h \left(T\left(Y^h\right) + \sum_i t_i X_i^h \right) = rG \Leftrightarrow \\ &F\left(L^1, L^2, G, E\right) - \sum_h \sum_t p_i X_i\left(\underline{Q}, B^h, L^h, G, E\right) = rG, \end{aligned} \qquad \textbf{eq. 2}$$

i.e. the sum of the tax revenues equals the price of public good provision.

Government's problem is to maximize the low ability type's utility given the level of type 2 utility. However, the government cannot distinguish the ability of a worker, thus the income tax must be defined by labour income. Because of this the government is restricted by a self-selection constraint, i.e. both household types must prefer to select the labour-taxation combination meant for them instead of mimicking the choice of the other type. Here it is concentrated on the situation, where the self-selection constraint binds only type 2 households[4]. The constraint can be written as

$$V^2\left(\underline{Q}, B^2, L^2, G, E\right) \geq \hat{V}^2\left(\underline{Q}, B^1, \Omega L^1, G, E\right), \qquad \textbf{eq. 3}$$

where the hat term $\hat{V}^2$ refers to the indirect utility of the mimicker. δ, λ, γ and μ are the lagrange multipliers for the Pareto constraint, the self-selection constraint, the government's budget constraint and the constraint defining the effect of the externality respectively. The Lagrange function of the government's optimization problem is

[3] Government's budget constraint can be rewritten with the help of the consumer's budget constraint:

$$\sum_h \left(T\left(Y^h\right) + \sum_t t_i X_i^h \right) = rG \rightarrow \sum_h \left(Y^h - B^h + \sum_t t_i X_i^h \right) = rG$$

$$\rightarrow \sum_h \left(Y^h - \sum_t \left(p_i + t_i\right) X_i + \sum_t t_i X_i^h \right) = rG,$$

and $\sum_h Y^h = F\left(L^1, L^2, G, E\right)$.

[4] The analysis would be analogous if the self-selection constraint bounded only low productivity households.

$$\begin{aligned}\Psi &= V^1\left(\underline{Q}, B^1, L^1, G, E\right) + \delta\left[V^2\left(\underline{Q}, B^2, L^2, G, E\right) - \overline{V}^2\right] \\ &+ \lambda\left[V^2\left(\underline{Q}, B^2, L^2, G, E\right) - \hat{V}^2\left(\underline{Q}, B^1, \Omega L^1, G, E\right)\right] \\ &+ \gamma\left[F\left(L^1, L^2, G, E\right) - \sum_h\sum_i p_i X_i^h\left(\underline{Q}, B^h, L^h, G, E\right) - rG\right] \\ &+ \mu\left[E - \sum_h X_d^h\left(\underline{Q}, B^h, L^h, G, E\right)\right].\end{aligned} \qquad \textbf{eq. 4}$$

The planner maximizes this function with respect of B^h, L^h, commodity tax rate t_j and E. The first order conditions with respect to these variables are

L¹: $$V_L^1 - \lambda\hat{V}_L^2\left(\Omega + L^1\frac{\partial\Omega}{\partial L^1}\right) + \gamma\left(w^1 - \sum_i p_i\frac{\partial X_i^1}{\partial L^1}\right) - \mu\frac{\partial X_d^1}{\partial L^1} = 0 \qquad \textbf{eq. 5}$$

B¹: $$V_B^1 - \lambda\hat{V}_B^2 - \gamma\sum_i p_i\frac{\partial X_i^1}{\partial B^1} - \mu\frac{\partial X_d^1}{\partial B^1} = 0 \qquad \textbf{eq. 6}$$

L²: $$(\delta + \lambda)V_L^2 - \lambda\hat{V}_L^2 L^1\frac{\partial\Omega}{\partial L^2} + \gamma\left(w^2 - \sum_i p_i\frac{\partial X_i^2}{\partial L^2}\right) - \mu\frac{\partial X_d^2}{\partial L^2} = 0 \qquad \textbf{eq. 7}$$

B²: $$(\delta + \lambda)V_B^2 - \gamma\sum_i p_i\frac{\partial X_i^2}{\partial B^2} - \mu\frac{\partial X_d^2}{\partial B^2} = 0 \qquad \textbf{eq. 8}$$

$\mathbf{t_j}$: $$V_q^1 + \delta V_q^2 + \lambda\left(V_q^2 - \hat{V}_q^2\right) - \gamma\sum_h\sum_i p_i\frac{\partial X_j^h}{\partial t_j} - \mu\sum_h\frac{\partial X_d^h}{\partial t_j} = 0 \qquad \textbf{eq. 9}$$

E: $$\begin{aligned}&V_E^1 + (\delta + \lambda)V_E^2 - \lambda\left(\hat{V}_E^2 + \hat{V}_L^2 L^1\frac{\partial\Omega}{\partial E}\right) \\ &+ \gamma\left(F_E - \sum_h\sum_i p_i\frac{\partial X_i^h}{\partial E}\right) + \mu\left(1 - \sum_h\frac{\partial X_d^h}{\partial E}\right) = 0,\end{aligned} \qquad \textbf{eq. 10}$$

where the subscripts L, B and E refer to the derivatives with the corresponding argument, and the subscript q denotes the derivative with respect to the price vector $\underline{Q}$.

3. Valuation of the externality

The shadow price, i.e. the valuation of the externality will be shown to be an essential part of our results. The form we use here is $\frac{\mu}{\gamma}$, the shadow price of the externality relative to government's tax revenue. To find an expression for the term, add and subtract $\lambda \hat{V}_B^2 \frac{V_E^1}{V_B^1}$ from equation 10 to get

$$\left(V_B^1 - \lambda \hat{V}_B^2\right)\frac{V_E^1}{V_B^1} + (\delta + \lambda)V_B^2 \frac{V_E^2}{V_B^2} - \lambda \hat{V}_B^2\left(\frac{\hat{V}_E^2}{\hat{V}_B^2} - \frac{V_E^1}{V_B^1}\right)$$
$$- \lambda \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial E} + \gamma\left(F_E - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial E}\right) + \mu\left(1 - \sum_h \frac{\partial X_d^h}{\partial E}\right) = 0. \qquad \textbf{eq. 11}$$

Let $MWP_{EB}^h = -\frac{V_E^h}{V_B^h}$ denote the marginal willingness to pay to avoid the externality. Plugging this into the previous equation yields

$$-\left(V_B^1 - \lambda \hat{V}_B^2\right)MWP_{EB}^1 - (\delta + \lambda)V_B^2 MWP_{EB}^2 + \lambda \hat{V}_B^2\left(M\hat{W}P_{EB}^2 - MWP_{EB}^1\right)$$
$$- \lambda \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial E} + \gamma\left(F_E - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial E}\right) + \mu\left(1 - \sum_h \frac{\partial X_d^h}{\partial E}\right) = 0. \qquad \textbf{eq. 12}$$

Substituting in the first order conditions 6 and 8 we get

$$-\left(\gamma \sum_t p_i \frac{\partial X_i^1}{\partial B^1} + \mu \frac{\partial X_d^1}{\partial B^1}\right)MWP_{EB}^1 - \left(\gamma \sum_t p_i \frac{\partial X_i^2}{\partial B^2} + \mu \frac{\partial X_d^2}{\partial B^2}\right)MWP_{EB}^2$$
$$+ \lambda \hat{V}_B^2\left(M\hat{W}P_{EB}^2 - MWP_{EB}^1\right) - \lambda \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial E} \qquad \textbf{eq. 13}$$
$$+ \gamma\left(F_E - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial E}\right) + \mu\left(1 - \sum_h \frac{\partial X_d^h}{\partial E}\right) = 0.$$

Next we use the Slutsky-type property of the conditional demand function to simplify the equation further. The conditional demand function x_i^h is obtained by minimizing the consumer's cost and keeping the utility constant. We know that the conditional demand function satisfies the property[5] $\frac{\partial X_i^h}{\partial E} = \frac{\partial x_i^h}{\partial E} - MWP_{EB}^h \frac{\partial X_i^h}{\partial B}$. It has also been shown in Edwards et al. (1994) that $\sum_i p_i \frac{\partial x_i^h}{\partial E} = MWP_{EB}^h - \sum_i t_i \frac{\partial x_i^h}{\partial E}$. Substituting these in and using a definition $\lambda^* = \frac{\lambda \hat{V}_B^2}{\gamma}$ equation 13 becomes

$$
\begin{aligned}
&-\frac{\mu}{\gamma} MWP_{EB}^1 \frac{\partial X_d^1}{\partial B^1} - \frac{\mu}{\gamma} MWP_{EB}^2 \frac{\partial X_d^2}{\partial B^2} + \lambda^* \left(M\hat{W}P_{EB}^2 - MWP_{EB}^1 \right) + F_E \\
&+ \sum_h \sum_i t_i \frac{\partial x_i^h}{\partial E} - \frac{\lambda}{\gamma} \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial E} - \sum_h MWP_{EB}^h + \frac{\mu}{\gamma} \left(1 - \sum_h \left(\frac{\partial x_d^h}{\partial E} - MWP_{EB}^h \frac{\partial X_d^h}{\partial B} \right) \right) = 0.
\end{aligned}
\qquad \textbf{eq. 14}
$$

The equation above simplifies further to

$$
\begin{aligned}
\frac{\mu}{\gamma} \left(1 - \sum_h \frac{\partial x_d^h}{\partial E} \right) &= \sum_h MWP_{EB}^h - \lambda^* \left(M\hat{W}P_{EB}^2 - MWP_{EB}^1 \right) \\
&- F_E + \lambda^* \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial E} - \sum_h \sum_i t_i \frac{\partial x_i^h}{\partial E}.
\end{aligned}
\qquad \textbf{eq. 15}
$$

And thus the shadow price can be written as

$$
\frac{\mu}{\gamma} = \sigma \left[\underbrace{\sum_h MWP_{EB}^h}_{term\ C_d} \underbrace{- \lambda^* \left(M\hat{W}P_{EB}^2 - MWP_{EB}^1 \right)}_{term\ C_i} \underbrace{- F_E}_{term\ P_d} \underbrace{+ \lambda^* \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial E}}_{term\ LM} \underbrace{- \sum_h \sum_i t_i \frac{\partial x_i^h}{\partial E}}_{term\ G_i} \right], \qquad \textbf{eq. 16}
$$

$$
where\, \sigma = \frac{1}{1 - \sum_h \frac{\partial x_d^h}{\partial E}}.
$$

The coefficient σ is an environmental feedback parameter. It has been shown that the feedback parameter must be positive to assure that the model is stable (Sandmo, 1980). The terms in brackets are divided into terms concerning consumers (*C*), producers (*P*), the labour markets (*LM*) and the government (*G*) and subscript *d* refers to direct effects and subscript *i* to indirect effects.

[5] Edwards et al. (1994) show this for a public good and the proof is analogous for a public bad as well.

The first term in the brackets, C_d, describes the direct harmful effect of the externality on consumers. The sign of the sum of the marginal willingnesses to pay to avoid the externality is positive. The shadow price is affected indirectly by the self-selection constraint C_i. Its sign depends on the difference in valuation of the externality between true type 1 and a mimicker. Marginal willingness to pay can be assumed to be the larger the more household has leisure (when environmental quality and leisure are complements[6]). Since mimicker is of the high ability type, he can do the work of type 1 in a shorter time than a true low ability type worker, and thus he has more leisure. Using this conclusion, the MWP_{EB} is larger for mimicker than for type 1 household and the term has a negative effect on the shadow price of the externality. This means that since mimickers are willing to pay more than true type 1 workers to avoid the externality, reducing the level of the externality increases the desirability of mimicking and thus forces the government to decrease the level of the income redistribution. Thus environmental objectives and aims regarding to income distribution seem to be in discrepancy with respect to this term[7].

In the production side the direct effect P_d increases the shadow price, as the derivative of the production function with respect to the externality was assumed to be negative. The indirect effect to the producer side comes from the labour market term LM as a result of wage adjustment[8]. λ^*, L^1 and $\hat{V}_B^2$ are positive. $\hat{V}_L^2$ is mimicker's marginal utility from labour, and it is negative. Thus the sign of LM term depends on the last part, the partial derivative of the wage rate with respect to the externality.

Since $\Omega = \dfrac{w^1}{w^2} = \dfrac{\partial F\left(L^1, L^2, E\right)/\partial L^1}{\partial F\left(L^1, L^2, E\right)/\partial L^2} = \dfrac{F_1\left(L^1, L^2, E\right)}{F_2\left(L^1, L^2, E\right)}$, the partial derivative is

[6] The opposite case, where environmental quality and leisure are substitutes, would go in an analogous way. However, this possibility seems less likely than complementarity of leisure and environmental quality.

[7] In the consumer side there are two indirect effects, one comes from the self-selection constraint and the other from the labour market effect as the wages adjust. It turns out that the sign of the labour market effect is most plausibly positive, so these two effects have opposite signs and as a whole the direction of the indirect effect in the consumer side remains ambiguous.

[8] In the labour market term the partial derivative of the wage ratio with respect to the externality Ω_E affects both consumers and producers.

$$\frac{\partial \Omega}{\partial E} = \Omega_E = \frac{F_{1E}F_2 - F_{2E}F_1}{(F_2)^2} = \frac{F_1}{F_2}\left(\frac{F_{1E}}{F_1} - \frac{F_{2E}}{F_2}\right) = \Omega\left(\frac{F_{1E}}{w^1} - \frac{F_{2E}}{w^2}\right)$$
$$= \Omega \frac{1}{E}\left(\xi_{1E} - \xi_{2E}\right), \qquad \textbf{eq. 17}$$

where the subscripts refer to the partial derivatives. Thus the sign of Ω_E is determined by ξ_{1E} and ξ_{2E}, the elasticities of wages with respect to the externality. Assuming pollution to have a negative effect on productivity, these elasticities are negative. Ω_E is negative, when $|\xi_{1E}| > |\xi_{2E}|$. This means that type 1 worker's productivity and wage increases more than type 2's productivity as the level of pollution is decreased[9]. If the elasticity for type 2 is greater (in absolute terms) then the sign of Ω_E is positive. It is plausible to think that the low ability workers could be more vulnerable to pollution than the high ability workers (for example because with lower income level they cannot afford to protect themselves from the pollution). Thus the externality is here assumed to have a negative effect on wage ratio. Assuming that Ω_E is negative means that the term *LM* is positive and it has an increasing effect on the shadow price.

Negative Ω_E means that when the government seeks to lessen the level of pollution, the wage ratio Ω rises. As Ω gets closer to 1, wage differences decrease. Smaller wage differences mitigate the self-selection constraint and allow the low ability type to earn more. This in turn leads to lower need for income transfers. Thus the taxation of a high ability type can be decreased and both types are better off. The income distribution target is in that case in accordance with environmental targets. If the effect of the externality on the wage ratio is positive, the government redistributing income and preferring environmental quality ends up in a trade off situation. Emphasizing the environmental target worsens the wage ratio, encourages mimicking and thus has on the other hand a negative effect on welfare.

The last term in equation 16, G_i refers to the externality's indirect effect on government's tax revenues. It tells how much government's tax revenues from commodity taxation change due to the externality affecting the demand for goods. When an increased pollution

[9] The reasoning is the same in the case where the effect of the externality on productivity is positive (although this kind of situation is hard to imagine). Ω_E is positive, if type 2 worker suffers more from the increased amount of the externality. The other possibilities are that $\xi_{1E} < 0$ and $\xi_{2E} > 0$, when Ω_E is always negative, and in the opposite case it is always positive.

level decreases the demand for goods, the tax revenue effect has a negative sign and term G_i increases the shadow price. In the opposite case the effect is of course the other way around.

When the level of the externality decreases the demand for other goods there will appear a double dividend. From the government's point of view it is advantageous to try to reduce the level of pollution because it would mean not only better environmental quality but also higher tax revenues from commodity taxes. Equivalently when the externality deters mimicking environmental policy leads to both less pollution and higher utility for workers as a result of mitigated self-selection constraint.

As a whole, the sign of the shadow price is ambiguous. Both of the direct effects increase the shadow price implying the harmfulness of the externality. The indirect effects are in turn less straight forward to interpret. In the consumer side the indirect effect of the self-selection constraint was assumed to be negative: aim to contribute the redistribution increases the shadow price and worsens the environmental quality. However, the labour market effect e.g. an indirect effect on both producers and consumers, is more plausible to believe to have a positive influence on the shadow price. This means that both environmental objectives and redistributional goals are in agreement with each other.

However, assuming that the externality is socially harmful would indicate that the shadow price must be positive. In the following discussion it will be assumed that the externality is harmful and its valuation is positive. On the other hand, it should be noted, that this is not the only conclusion that can be made. If the presence of the externality discourages mimicking such that term C_i is sufficiently large and/or it increases the tax revenues by boosting the demand of goods, it is possible that the shadow price is negative. This implies that the externality would in this case actually be socially beneficial.

Assume now as a special case of the former analysis that the externality affects only consumers, i.e. pollution does not influence the production at all. Term P_d in equation 16 would become zero, and the shadow price would reduce to

$$\frac{\mu}{\gamma}=\sigma\left[\sum_h MWP_{EB}^h-\lambda*\left(M\hat{W}P_{EB}^2-MWP_{EB}^1\right)+\lambda*\frac{\hat{V}_L^2}{\hat{V}_B^2}L^1\frac{\partial\Omega}{\partial E}-\sum_h\sum_i t_i\frac{\partial x_i^h}{\partial E}\right]. \qquad \textbf{eq. 18}$$

The sign of the shadow price remains ambiguous, as the first and the third terms are positive, second term is negative and the sign of the tax revenue term depends on the externality's effect on demand of goods. As earlier, the term referring to the difference between mimicker's and true low productivity type is negative implying inconsistency between environmental and distributional objectives.

Another special case is to assume that the externality affects only in firms' production but not the consumer's utilities[10]. Now terms C_d and C_i are zero and the valuation of the externality can be written as

$$\frac{\mu}{\gamma}=\sigma\left[-F_E+\lambda*\frac{\hat{V}_L^2}{\hat{V}_B^2}L^1\frac{\partial\Omega}{\partial E}-\sum_h\sum_i t_i\frac{\partial x_i^h}{\partial E}\right]. \qquad \textbf{eq. 19}$$

The two first terms were assumed to be positive and if the externality decreases the demand for goods, also the last term has an increasing effect on the shadow price. In this case the shadow price would be unambiguously positive implying the harmfulness of the externality. Investing on reducing the amount of the externality is profitable as improving environmental quality also promotes distributional aims.

4. Effective marginal tax rates

In this model the tax system consists of a direct income tax and an indirect commodity tax. The total tax paid by a worker is the sum of these two taxes $\tau(Y)=T(Y)+\sum_t t_i X_i$. The effective marginal tax rate (*MTR*) can be found by differentiating total taxes with respect to income *Y*. The differentiation yields[11]

[10] However, through the labour market term the externality would affect consumers also in this case as a result of endogenous wages.

[11] Here the results $B=Y-T(Y)\rightarrow\frac{\partial B}{\partial Y}=1-T'$ and $Y=wL\rightarrow\frac{\partial L}{\partial Y}=\frac{1}{w}$ are used.

$$\tau'(Y) = T' + \sum_i t_i \left(\frac{\partial X^i}{\partial B}(1 - T') + \frac{\partial X_i}{\partial L}\frac{1}{w^i} \right). \qquad \textbf{eq. 20}$$

The marginal income tax rate is derived as a result of households' optimization problem where the indirect demand function $V^h(Q, B, L, G, E)$ is maximized subject to the budget equation $B^h = Y^h - T(Y^h)$. The resulting condition is

$$T' = 1 + \frac{V_L}{wV_B} \rightarrow 1 - T' = -\frac{V_L}{wV_B}. \qquad \textbf{eq. 21}$$

Substituting the previous condition into equation 20 the effective *MTR* for type *i* becomes

$$\begin{aligned} \tau'^i &= 1 + \frac{V_L}{w^i V_B} + \sum_i t_i \left(-\frac{\partial X_i}{\partial B}\frac{V_L}{w^i V_B} + \frac{\partial X_i}{\partial L}\frac{1}{w^i} \right) \\ &= 1 + \frac{V_L}{w^i V_B}\left[1 - \sum_i t_i \frac{\partial X_i}{\partial B} \right] + \frac{1}{w^i}\sum_i t_i \frac{\partial X_i}{\partial L}. \end{aligned} \qquad \textbf{eq. 22}$$

The commodity tax rate t_i is the difference between the producer price p_i and the consumer price q_i: $q_i = t_i + p_i$. The conditional demand functions X_i satisfy the adding-up conditions $\sum_i q_i \frac{\partial X_i}{\partial B} = 1$ and $\sum_i q_i \frac{\partial X_i}{\partial Y} = 0$. Since $Y = wL$, $\frac{\partial X_i}{\partial Y} = \frac{\partial X_i}{\partial L}\frac{1}{w}$ and the second adding up condition can be written as $\frac{1}{w}\sum_i q_i \frac{\partial X_i}{\partial L} = 0$. Taking these properties into account equation 22 becomes

$$\begin{aligned} \tau'^i &= 1 + \frac{V_L}{w^i V_B}\sum_i p_i \frac{\partial X_i}{\partial B} - \frac{1}{w^i}\sum_i p_i \frac{\partial X_i}{\partial L} \\ &= \frac{1}{w^i}\left[w^i + \frac{V_L}{V_B}\sum_i p_i \frac{\partial X_i}{\partial B} - \sum_i p_i \frac{\partial X_i}{\partial L} \right]. \end{aligned} \qquad \textbf{eq. 23}$$

To find an expression for the second term in the right hand side divide equation 7 by equation 8. By rearranging the expression we get

$$\frac{V_L^2}{V_B^2}\sum_i p_i \frac{\partial X_i^2}{\partial B^2} = -\left(w^2 - \sum_i p_i \frac{\partial X_i^2}{\partial L^2} \right) + \frac{\mu}{\gamma}\left(\frac{\partial X_d^2}{\partial L^2} - \frac{V_L^2}{V_B^2}\frac{\partial X_d^2}{\partial B^2} \right) + \lambda \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial L^2}. \qquad \textbf{eq. 24}$$

Plugging this back into equation 23 the effective marginal tax rate for type 2 becomes

$$\tau'^2 = \frac{1}{w^2}\left(\underbrace{\frac{\mu}{\gamma}\left(\frac{\partial X_d^2}{\partial L^2} - \frac{V_L^2}{V_B^2}\frac{\partial X_d^2}{\partial B^2} \right)}_{term\ A_2} + \underbrace{\lambda * \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial L^2}}_{term\ B_2} \right). \qquad \textbf{eq. 25}$$

The effective marginal tax rate of type 2 consists of two terms A_2 and B_2 (subscript refers to worker's type). Assuming that the shadow price of the externality is positive and that $\frac{V_L^i}{V_B^i}$ is negative, term A_2 is positive when the dirty good is a normal good and a substitute with leisure[12]. This term describes the effect of the externality: if there were no externality, the shadow price would be zero and the whole term would vanish. The term B_2 captures the effect of endogenous wages. Since $\frac{\partial \Omega}{\partial L^2}$ is positive and $\frac{\hat{V}_L^2}{\hat{V}_B^2}$ is negative, the sign of term B_2 is negative. This implies that letting the wage rates adjust endogenously decreases the marginal tax rate of a high ability person.

Equivalently the effective marginal tax rate for type 1 can be determined by calculating an expression for the second term in equation 23. Dividing the first order conditions, equation 5 by equation 6, and rearranging yields

$$\begin{aligned} \frac{V_L^1}{V_B^1}\sum_i p_i \frac{\partial X_i^1}{\partial B^1} &= -\left(w^1 - \sum_i p_i \frac{\partial X_i^1}{\partial L^1} \right) + \frac{\mu}{\gamma}\left(\frac{\partial X_d^1}{\partial L^1} - \frac{V_L^1}{V_B^1}\frac{\partial X_d^1}{\partial B^1} \right) \\ &\quad + \frac{\lambda}{\gamma}\hat{V}_B^2\left(\frac{\hat{V}_L^2}{\hat{V}_B^2}\left(\Omega + L^1\frac{\partial \Omega}{\partial L^1} \right) - \frac{V_L^1}{V_B^1} \right). \end{aligned} \qquad \textbf{eq. 26}$$

Plugging this into equation 23 gives the effective marginal tax rate of type 1:

$$\tau'^1 = \frac{1}{w^1}\left[\frac{\mu}{\gamma}\left(\frac{\partial X_d^1}{\partial L^1} - \frac{V_L^1}{V_B^1}\frac{\partial X_d^1}{\partial B^1} \right) + \lambda * \left(\frac{\hat{V}_L^2}{\hat{V}_B^2}\Omega - \frac{V_L^1}{V_B^1} \right) + \lambda * \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial L^1} \right]. \qquad \textbf{eq. 27}$$

[12] When the dirty good is normal and complement with leisure, the first derivative in term A_2 is negative and the second part is positive, and the sign of the term remains ambiguous.

The first term in the brackets can be rewritten by adding and subtracting a term $\frac{\lambda}{\gamma}\hat{V}_L^2(1-\Omega)$ as

$$\lambda*\left(\frac{\hat{V}_L^2}{\hat{V}_B^2}\Omega-\frac{V_L^1}{V_B^1}\right)=\frac{\lambda}{\gamma}\left(\hat{V}_L^2\Omega-\frac{V_L^1}{V_B^1}\hat{V}_B^2+\hat{V}_L^2(1-\Omega)-\hat{V}_L^2(1-\Omega)\right)$$
$$=\frac{\lambda}{\gamma}\left(-\frac{V_L^1}{V_B^1}\hat{V}_B^2+\hat{V}_L^2-\hat{V}_L^2+\hat{V}_L^2\Omega\right)=\lambda*\left(\frac{\hat{V}_L^2}{\hat{V}_B^2}-\frac{V_L^1}{V_B^1}\right)-\lambda*\frac{\hat{V}_L^2}{\hat{V}_B^2}(1-\Omega). \qquad \textbf{eq. 28}$$

So the effective *MTR* for type 1 becomes

$$\tau'^1=\frac{1}{w^1}\left[\underbrace{\frac{\mu}{\gamma}\left(\frac{\partial X_d^1}{\partial L^1}-\frac{V_L^1}{V_B^1}\frac{\partial X_d^1}{\partial B^1}\right)}_{term\ A_1}+\underbrace{\lambda*\frac{\hat{V}_L^2}{\hat{V}_B^2}L^1\frac{\partial\Omega}{\partial L^1}}_{term\ B_1}+\underbrace{\lambda*\left(\frac{\hat{V}_L^2}{\hat{V}_B^2}-\frac{V_L^1}{V_B^1}\right)}_{term\ C_1}-\underbrace{\lambda*\frac{\hat{V}_L^2}{\hat{V}_B^2}(1-\Omega)}_{term\ D_1}\right]. \qquad \textbf{eq. 29}$$

Now the effective marginal tax rate for the low ability person is determined by four terms. The two first terms, A_1 and B_1 correspond to the ones of high ability person, the only difference is, that here $\frac{\partial\Omega}{\partial L^1}$ is negative and thus term B_1 is positive. Term C_1 defines the effect of the self-selection constraint. By agent monotonicity[13] this term is positive. Term D_1 is an outcome of the endogenous wages and it is positive. Thus the effective marginal tax rate of the low ability type is strictly positive when the dirty good is normal and a substitute with leisure.

In conclusion from equations 25 and 29 it seems like the effect of endogenous wages decreases the effective MTR of the more productive worker (terms B_1 and D_1) and raises the effective MTR of the low productive worker (term B_2). If there were no externality, terms A_1 and A_2 would disappear from the equations of effective MTRs. In other words, when the dirty good is a normal good and a substitute with leisure, presence of externalities increases effective MTR. This result is in accordance with Pirttilä and Tuomala (1997) with

[13] The agent monotonicity condition implies that $\frac{V_L^i}{V_B^i}$ decreases with productivity.

the distinction that here it needs to determined whether the dirty good is a substitute or a complement with leisure in order to find out the sign of the partial derivative $\frac{\partial X_d^h}{\partial L^h}$.

However, these results of marginal tax rates don't give enough information to say anything about the income tax level. The income tax may rise or fall due to the externality in the endogenous wages' framework.

5. Commodity Taxation

The optimal commodity taxation rule can be derived from the first order condition with respect to the tax rate t_j. As a result of using Roy's identity equation 9 can be written as

$$-V_B^1 X_j^1 - (\delta + \lambda) V_B^2 X_j^2 - \lambda \hat{V}_B^2 \hat{X}_j^2 - \gamma \sum_h \sum_i p_i \frac{\partial X_j^h}{\partial t_j} - \mu \sum_h \frac{\partial X_d^h}{\partial t_j} = 0. \qquad \textbf{eq. 30}$$

By differentiating consumer's budget constraint $\sum_i q_i X_i = B_i$ with respect to q_j we get a condition $\sum_i q_i \frac{\partial X_i}{\partial t_j} + X_j = 0$. Plugging this in and doing some rearrangements yields

$$\begin{aligned} &\phi_j + \sum_h \sum_t t_i \frac{\partial X_j^h}{\partial t_j} - \frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial t_j} = 0, \\ &where\ \phi_j = -\sum_h X_j^h + \frac{1}{\gamma}\left[V_B^1 X_j^1 + (\delta + \lambda) V_B^2 X_j^2 - \lambda \hat{V}_B^2 \hat{X}_j^2\right]. \end{aligned} \qquad \textbf{eq. 31}$$

Term ϕ_j includes the elements from equation 30 not referring to the externality. Pirttilä and Tuomala (1997) define this term as the public finance part of commodity tax t_j. This part of commodity taxation thus remains unchanged even if the externality disappears.

Equation 31 can also be given in a matrix form:

$$\begin{bmatrix} \sum_h \frac{\partial X_c^h}{\partial q_c} & \sum_h \frac{\partial X_d^h}{\partial q_c} \\ \sum_h \frac{\partial X_c^h}{\partial q_d} & \sum_h \frac{\partial X_d^h}{\partial q_d} \end{bmatrix} \begin{bmatrix} t_c \\ t_d \end{bmatrix} = \begin{bmatrix} \phi_c + \frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial q_c} \\ \phi_d + \frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial q_d} \end{bmatrix}. \qquad \textbf{eq. 32}$$

Denoting the determinant of the coefficient matrix in the left by J and utilizing Cramer's rule we can solve commodity taxes separately for goods c and d:

$$t_c = \frac{1}{J}\sum_h \left(\phi_c \frac{\partial X_d^h}{\partial q_d} - \phi_d \frac{\partial X_d^h}{\partial q_c} \right) \qquad \textbf{eq. 33}$$

$$t_d = \frac{1}{J}\sum_h \left(\phi_c \frac{\partial X_d^h}{\partial q_d} - \phi_d \frac{\partial X_d^h}{\partial q_c} \right) + \frac{\mu}{\gamma}. \qquad \textbf{eq. 34}$$

These equations are exactly the same as in the case of exogenous wages. Dixit's (1985) general result of "principle of targeting" continues to hold also under the assumption of endogenous wages. The part of the commodity tax indicating the harmfulness of the externality $\frac{\mu}{\gamma}$ appears only in the tax rate of the dirty good. The more specific result is Sandmo's additivity property, which states that the term connected to the harmful externality appears additively in the tax rate of the dirty good.

The assumption of the endogeneity of wages doesn't seem to affect commodity taxation at all. This outcome is not a surprise since the reason why endogeneity changed the equations of the shadow price or marginal tax rates was a result of the nonlinear technology and self-selection constraints, which do not appear in commodity taxation.

6. Public Expenditure

The condition for an optimal production of the public good can be derived by taking a first order condition from equation 4 with respect to G:

$$\frac{\partial \Psi}{\partial G} = V_G^1 + (\delta + \lambda) V_G^2 - \lambda\left(\hat{V}_G^2 + \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial G} \right) + \gamma\left(F_G - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial G} + r \right) - \mu \sum_h \frac{\partial X_d^h}{\partial G}. \quad \textbf{eq. 35}$$

The manipulation goes exactly as in determining the shadow price for the externality. First the term $\lambda \hat{V}_B^2 \frac{V_E^1}{V_B^1}$ is added and subtracted in equation 35. After some rearranging and using a definition of marginal rate of substitution $MRS_{GB}^h = \frac{V_G^h}{V_B^h}$ we get

$$\left(V_B^1 - \lambda V_B^2\right)MRS_{GB}^1 + (\delta + \lambda)V_B^2 MRS_{GB}^2 - \lambda \hat{V}_B^2\left(M\hat{R}S_{GB}^2 - MRS_{GB}^1\right) - \lambda \hat{V}_L^2 L^1 \frac{\partial \Omega}{\partial G} + \left(F_G - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial G} + r\right) - \mu \sum_h \frac{\partial X_d^h}{\partial G} = 0. \qquad \textbf{eq. 36}$$

Substituting the first order conditions 6 and 8 in equation 36 and dividing it by γ yields

$$\sum_h \sum_i p_i \frac{\partial X_i^h}{\partial B} MRS_{GB}^h + \frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial B} MRS_{GB}^h - \lambda * \left(M\hat{R}S_{GB}^2 - MRS_{GB}^1\right) - \lambda * \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial G} + F_G - \sum_h \sum_i p_i \frac{\partial X_i^h}{\partial G} - r - \frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial G} = 0. \qquad \textbf{eq. 37}$$

Using similar properties as earlier, $\frac{\partial X_i^h}{\partial G} = \frac{\partial x_i^h}{\partial G} - MWP_{GB}^h \frac{\partial X_i^h}{\partial B}$ and $\sum_i p_i \frac{\partial x_i^h}{\partial G} = MWP_{GB}^h - \sum_i t_i \frac{\partial x_i^h}{\partial G}$, the equation above becomes

$$\sum_h MRS_{GB}^h = \underbrace{r + \lambda * \left(M\hat{R}S_{GB}^2 - MRS_{GB}^1\right)}_{\textit{term } A_G} + \underbrace{F_G}_{\textit{term } B_G} - \underbrace{\sum_h \sum_i t_i \frac{\partial X_i^h}{\partial G}}_{\textit{term } C_G} + \underbrace{\frac{\mu}{\gamma} \sum_h \frac{\partial X_d^h}{\partial G}}_{\textit{term } D_G} \underbrace{- \lambda * \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial G}}_{\textit{term } E_G}. \qquad \textbf{eq. 38}$$

Equation 38 defines the optimal level of the public production. The first term in the right hand side is the price of the public good (r). It corresponds to original Samuelson rule for the optimal public provision. Second term A_G captures the effect of the self-selection constraint. The desirability of mimicking can be affected by regulating the public provision[14]. Term B_G is the direct effect for production, which was assumed to be positive.

[14] When the true type 1 values the public good more than high ability type, by increasing public provision the self-selection constraint can be mitigated (Boadway ja Keen, 1993).

Term C_G defines how public provision affects government's tax revenues. The effect of the externality is described in term D_G. When the shadow price of the externality is assumed to be positive, this effect is positive, if the public good and the dirty good are complements. In other words, raising the level of public production increases the consumption of the dirty good and thus worsens the environmental quality[15] (Pirttilä and Tuomala, 1997). The last term E_G denotes the effect of the endogenous wages. The level of the optimal public provision is increased, when $\frac{\partial \Omega}{\partial G}$ is positive, i.e. when an increase in G raises the wage ratio Ω and thus decreases the difference between wages. In this case, public provision can be used as a part of redistribution policy.

Terms C_G and D_G can be written with the help of the shadow price as $\sum_h \left[t_c \frac{\partial X_c^h}{\partial G} + \left(t_d - t_d^P\right) \sum_h \frac{\partial X_d^h}{\partial G} \right]$. This term indicates that when public and private goods are complements, an increase in public provision leads to a larger consumption of private goods and thus increases tax revenues from commodity taxation. However, since the dirty commodity creates a harmful externality, the public finance part of the commodity tax, i.e. the cost of internalizing the externality, needs to be subtracted from tax revenues. After using this term, equation 38 can be written as

$$\sum_h MRS_{GB}^h =$$

$$\underbrace{r + \lambda * \left(M\hat{R}S_{GB}^2 - MRS_{GB}^1\right)}_{term\ A'_G} + \underbrace{F_G}_{term\ B'_G} \underbrace{- \sum_h \left[t_c \frac{\partial X_c^h}{\partial G} + \left(t_d - t_d^P\right) \sum_h \frac{\partial X_d^h}{\partial G} \right]}_{term\ C'_G} \underbrace{-\lambda * \frac{\hat{V}_L^2}{\hat{V}_B^2} L^1 \frac{\partial \Omega}{\partial G}}_{term\ E'_G}. \qquad \textbf{eq. 39}$$

This equation implies that in addition to the ordinary Samuelson rule of optimal public provision, there are extra terms affecting the optimal level. The result is in accordance with the study of Bovenberg and van der Ploeg (1994) and Pirttilä and Tuomala (1997). The term E'_G in equation 39 is added to this analysis as a result of the assumption of endogenous wages.

[15] In the opposite case, when public good and dirty good are substitutes, clearly it would be optimal to increase the level of public production and replace part of dirty good's consumption with a public good which is not creating an externality.

Consider now a special case where the utility function is separable between goods and leisure. This assumption would make term A'_G disappear because now mimicker and the true type 1 valuate the public good exactly the same. If we make a further assumption that utility is also separable between private goods and public good, also term C'_G would cancel out to zero. However, even these assumptions are not enough to make the Samuelson rule to apply. There are still left terms B'_G indicating the effect of production side and E'_G as a result of endogenous wages.

7. Conclusions

This paper analyses the optimal tax policy in the presence of a harmful externality under an assumption of endogenous wages. The government, restricted by a self-selection constraint, has both environmental and redistributional objectives. An essential term defining the optimal taxes is the social valuation of the externality. This valuation consists of 1) direct and indirect effects on consumers depending on marginal willingnesses to pay to avoid the externality, 2) direct effect on producers as a result of the externality's influence on production 3) the labour market effect influencing indirectly both producers and consumers through the adjustment of the wage ratio, 4) tax revenue change affecting the government and 5) an environmental feedback parameter. Both direct effects increase the valuation of the externality implying its harmfulness for both consumers and producers. The self-selection effect has a negative effect on the social valuation of the externality when environmental quality and leisure are complements. This means that the environmental deterioration discourages mimicking and promotes government's redistributional goals. An interesting result is that in the consumer side the direct effect and the self-selection effect have opposite signs, i.e. environmental and redistributional objectives are inconsistent whereas in producer side the direct effect and the labour market effect have both positive signs denoting that contributing environmental aims promotes also redistribution.

The effective marginal tax rates look pretty much the same as in the case of exogenous wages. Introducing the externality increases both types' effective marginal tax rates when the dirty good is a normal good and a substitute with leisure. The extra term indicating the effect of the endogenous wages decreases the effective marginal tax rate of the high productivity worker and raises the one of the low productivity worker. In the optimal commodity taxation Dixit's principle of targeting continues to hold: only the dirty good's tax includes the externality internalizing part, which is equal to the social valuation of the externality and enters additively the tax rate.

The next step is to widen this analysis by assuming that the low productivity type workers have a different vulnerability with respect to the externality. This assumption will give yet another dimension to analyze the externality's effect on optimal taxation. Also determination of the generality of principle of targeting is an interesting area of further research.

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